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Phase-Space Decoherence: a comparison between Consistent Histories and Environment Induced Superselection

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Abstract

We examine the decoherence properties of a quantum open system as modeled by a quantum optical system in the Markov regime. We look for decoherence in both the Environment Induced Superselection (EIS) and Consistent Histories (CH) frameworks. We propose a general measure of the coherence of the reduced density matrix and find that EIS decoherence occurs in a number of bases for this model. The degree of “diagonality” achieved increases with bath temperature. We evaluate the Decoherence Functional of Consistent Histories for coarse grained phase space two-time projected histories. Using the measures proposed by Dowker and Halliwell we find that the consistency of the histories improves with increasing bath temperature, time and final grain size and decreases with initial grain size. The peaking increases with increasing grain size and decreases with increasing bath temperature. Adopting the above proposed measure of “coherence” to the Decoherence Functional gives similar results. The results agree in general with expectations while the anomalous dependence of the consistency on the initial grain size is discussed.

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I. INTRODUCTION

The emergence of classical dynamics from the underlying quantum behaviour still continues to be one of the foremost questions in physics today. Two ingredients essential to this transition are (i) the establishing of suitable “classical” correlations between appropriate coordinates and their conjugate momenta (ie. classical equations of motion) and (ii) the suppression or destruction of quantum correlations. The need for the former is self-evident while the latter is necessary to explain the general absence of macroscopic quantum superpositions. A paradigm which primarily addresses (ii) and which attempts to describe this transition is *Decoherence*. In this school of thought, Decoherence is the general term which describes a dynamical process whereby the quantum correlations in the system in question are diminished or destroyed. These processes can be split into those which deviate from ordinary quantum mechanics, eg. GRW theory and quantum mechanical spontaneous localisation QMSL [2], and those processes which stay within the confines of quantum mechanics. We will only consider the latter in this paper. The two primary paradigms which attempt to describe the transition from quantum to classical using pure quantum mechanics are Environmental Induced Superselection (EIS) and Consistent Histories (CH) [3,4]. Although much has been written concerning these two theories, calculations comparing the two in non-trivial models are lacking. Even in relatively simple models the computations quickly become lengthy and difficult to interpret. In this paper we examine the dynamics of a simple quantum optical system from the viewpoints of both EIS and CH. Upon comparison we find that they generally agree, that is, if EIS occurs in a particular basis then histories in that basis become more consistent.

In section II we derive the general solution to the quantum optical master equation in the Markov regime for a system comprising of an harmonic oscillator linearly coupled to an infinite bath of harmonic oscillators with the usual quantum optical approximations. We solve for the evolution of the reduced density matrix of the system in the position, momentum, number and coherent state bases. In section III we discuss EIS in these bases using a general measure of “diagonality” which quantifies the peaking of the density matrix about the diagonal. This measure is sensitive to the coherence of the system state. We solve for the general behaviour of this measure in the position basis for an arbitrary initial system state and examine the particular case of a mixture of two coherent states. We find that EIS occurs to some extent in all of the above bases with $\rho_{reduced}$ becoming more “diagonal” with time and increasing bath temperature.

In section IV we calculate the decoherence functional of CH for two-time projected coarse-grained histories on phase-space. As in all previous calculations, we use quasi-projectors, that is, operators which are complete but are not exclusive, to define the coarse-grained histories on phase-space. We then compute the degree of consistency and peaking of the decoherence functional using measures advanced by Dowker and Halliwell [5]. We find that the consistency and peaking behave as expected except for an anomalous *decrease* in consistency with increased initial grain size. This behaviour is also found in all previous calculations. We also adopt the above proposed measure for diagonality in the EIS framework to quantify the “coherence” of the decoherence functional. We find a similar dependence of this measure on the model parameters to that displayed by the the Dowker-Halliwell measure. In the conclusion we discuss the results of the EIS and CH calculations and the anomalous

dependence of the consistency in the CH picture on the initial graining. We also discuss the legitimacy of using the “coherence” measure in the CH picture. We argue that in the case of a set of many alternatives, this “coherence” measure gives a good indication that the probability sum rule is met to the given degree.

Before examining the dynamics of the quantum optical master equation we first make a few remarks concerning both the EIS and CH scenarios.

A. Environment Induced Superselection

Environmentally Induced Superselection was primarily developed by Zurek in the early 1980’s [6] in relation to quantum non-demolition measurements of gravitational wave antennas. Since then, the theory has steadily evolved to encompass more complicated and realistic models. Essentially, the system in question is coupled to a large external bath via a particular interaction with Hamiltonian H_{int} . If the interaction is such that the reduced density matrix of the system becomes approximately diagonal in a particular basis in a time much less than the relaxation time then we say EIS has occurred. The process is aimed primarily at removing the quantum coherence from the system. The system’s reduced density matrix in this particular basis is approximately diagonal and is given a purely ignorance interpretation. That is, the system is now in one of the states on the diagonal but we do not know which one.

A number of calculations have shown that in some cases EIS is very successful and can damp away the quantum superpositions very quickly indeed [7,3]. Other models have shown however, that the spectral properties of environment play a significant role in determining the decoherence time [8,9]. In this paper we will work with the standard ohmic environment.

It is also the case that the initial conceptions of EIS have changed. The concept of a pointer basis $|\Lambda\rangle$, the particular basis in which the reduced density matrix becomes diagonal, strictly holds only in those models for which the interaction Hamiltonian commutes with the number operator of the pointer basis $\tilde{A}|\Lambda\rangle = \Lambda|\Lambda\rangle$. For more realistic cases and for the models treated in this paper, no exact pointer basis exist. Work on a suitable generalization of the concept of a pointer basis, that is, a system basis which is least effected by the interaction with the bath, is currently being pursued [10,11]. Also, there does not appear to be a general concrete description concerning the “diagonality” of a reduced density matrix in a particular basis. Some measures have been advanced for gaussian states. For particular non-gaussian states a fringe visibility measure based on the appearance of the Wigner function has been proposed [9]. This however, appears to quantify the amount of interference in the state rather than the coherence. EIS is the process whereby the reduced state of the system deviates more and more from a description as a single ray in Hilbert space. Such a process reduces the interferences present in the state. However, pure states, with unitary evolution, can display little interference. Such processes do not display EIS. With this in mind we propose and investigate a measure of the “diagonality” of the state in a number of bases in a quantum optical model. We find that the coherence of the reduced density matrix of the system decays and gives clear evidence of EIS in a number of bases. However, the degree of diagonality depends on the basis chosen. It also depends on the model parameters in an expected manner, becoming more diagonal with increasing bath temperature.

We have not examined the attainment of (i) above in the EIS framework. This is because there appears to be some dispute concerning the quantification of classical correlations in the state. Previously, some have examined the Wigner function of the state for peaks [12]. However, the Wigner function is not a true probability distribution and does not correspond to the results of any conceivable measurement on the system. The Q function of quantum optics *does* correspond to the result of a particular measurement scheme [13]. More generally, there exist true probability distribution functions corresponding to more general measurements [14]. To establish the existence and to quantify “classical” correlations one should examine such distributions. Work towards this goal is in progress [15].

Finally, the ignorance interpretation of the resulting diagonal reduced density matrix seems part and parcel of EIS. This feature of the framework has surprisingly not attracted much attention in the literature. If EIS is to describe a truly ontological reality then Zurek’s arguments may not suffice [16]. It is clearly capable of describing an empirical reality [16] but are it’s aspirations higher than just such a description?

B. Consistent Histories

In order not to confuse the reader with various different meanings of the term decoherence, we will refrain from the more usual nomenclature of “Decohering Histories” and use the more descriptive terminology – “Consistent Histories”. Consistent Histories was initially researched by Griffiths and Omnés [17,18] and later on independently rediscovered and greatly expanded on by Gell-Mann and Hartle [19]. This paradigm is more ambitious than that of EIS as it attempts to describe the emergence of the complete quasi-classical world of familiar experience from the underlying quantum world. Central to this framework is the notion of coarse-grained histories, each with an assigned probability. A primary goal is the identification of complete sets of coarse-grained histories which exactly satisfy the probability sum rules. Although much has been written concerning this theory, there are only a few calculations on realistic models [5,4]. Quantum optics is an avenue where both quantum and classical behaviour are accessible to the experimentalist. We will in particular look at coarse-grained histories on phase-space itself. Previous calculations have concentrated on position projected histories with momenta information gained through time-of-flight. In this paper we look for consistent histories in *both* position and momentum. We also note that the interpretation of the diagonal decoherence functional is typically a relative state interpretation and thus the interpretational problems concerning which history is realized are moot (however see [16]).

II. QUANTUM OPTICAL MASTER EQUATION

We begin by looking at the optical master equation for a system linearly coupled via the usual quantum optical coupling to a bath of harmonic oscillators [20,21]. For completeness and to relate this to earlier work let us first consider a system with a Hamiltonian

$$H/\hbar = \omega(t) a^\dagger a + f_1(t) a + f_1^*(t) a^\dagger + f_2(t) a^2 + f_2^*(t) a^{\dagger 2} . \quad (2.1)$$

In the quantum optical regime we can write the master equation for the reduced density matrix of the system as [20]

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \Lambda\rho = \mathcal{L}\rho \quad , \quad (2.2)$$

$$\Lambda\rho = \frac{\gamma(\bar{n}+1)}{2} \{[a, \rho a^\dagger] + [a\rho, a^\dagger]\} + \frac{\gamma\bar{n}}{2} \{[a^\dagger, \rho a] + [a^\dagger\rho, a]\} \quad . \quad (2.3)$$

This may be converted into a partial differential equation for a Quasi-Distribution Function (QDF), $W(\alpha, \alpha^*, s)$ (or $W(\alpha, s)$ for short) where [22]

$$W(\alpha, \alpha^*, s, t) = \frac{1}{\pi} \int \chi(\xi, s, t) e^{\alpha\xi^* - \alpha^*\xi} d^2\xi \quad , \quad (2.4)$$

$$\chi(\xi, s, t) = \text{Tr}[\rho(t)D(\xi, s)] \quad , \quad (2.5)$$

$$D(\xi, s) = \exp(\xi a^\dagger - \xi^* a + \frac{1}{2}s|\xi|^2) \quad , \quad (2.6)$$

where we have the normalization $\int W(\alpha, s, t) d^2\alpha/\pi = 1$. The family of distribution functions $W(\alpha, s, t)$ encompasses the more familiar distributions. For $s = 1$, $W(\alpha, +1, t) = P(\alpha, t)$ is the P distribution of Quantum Optics. For $s = 0$, $W(\alpha, 0, t) = W(\alpha, t)$ is the familiar Wigner distribution while for $s = -1$, $W(\alpha, -1, t) = Q(\alpha, t)$ is the Q or Husimi distribution. The different distributions correspond to different operator orderings. From (2.2) we can derive a partial differential equation for both $W(\alpha, s)$ and $\chi(\xi, s)$ from first principles using equations (2.4, 2.5, 2.6). However, using the general rules derived in Vogel and Risken [23] we can easily obtain

$$\begin{aligned} \dot{W}(\alpha, s, t) = & \\ & -\frac{i}{\hbar} \{ [f_1 + 2\alpha f_2 + \alpha^* \omega] \partial_{\alpha^*} - [f_1^* + 2\alpha^* f_2^* + \alpha \omega] \partial_\alpha - s [f_2 \partial_{\alpha\alpha^*}^2 - f_2^* \partial_{\alpha\alpha^*}^2] \} W(\alpha, s, t) \end{aligned} \quad (2.7)$$

$$+ \frac{\gamma}{2} \{ \partial_\alpha \alpha + \partial_{\alpha^*} \alpha^* + [2\bar{n} + 1 - s] \partial_{\alpha\alpha^*}^2 \} W(\alpha, s, t) \quad , \quad (2.8)$$

$$\begin{aligned} \dot{\chi}(\xi, s, t) = & \\ & -\frac{i}{\hbar} \{ \omega [\xi^* \partial_{\xi^*} - \xi \partial_\xi] - [\xi f_1 + \xi^* f_1^*] - s [\xi^2 f_2 - \xi^{*2} f_2^*] + 2 [f_2 \partial_{\xi^*} - f_2^* \partial_\xi] \} \chi(\xi, s, t) \end{aligned} \quad (2.9)$$

$$+ \frac{\gamma}{2} \{ (s - 1 - 2\bar{n}) \xi \xi^* - \xi^* \partial_{\xi^*} - \xi \partial_\xi \} \chi(\xi, s, t) \quad , \quad (2.10)$$

where we have dropped the t dependence of f_i and ω . We see that the PDE for χ is first order. The general solution for this model was treated in [21]. We will instead specialize to the case of a single harmonic oscillator with constant frequency linearly coupled to the bath oscillators. Going to the interaction picture we get

$$\dot{W}(\alpha, s, t) = \frac{\gamma}{2} \{ \partial_\alpha \alpha + \partial_{\alpha^*} \alpha^* + [2\bar{n} + 1 - s] \partial_{\alpha\alpha^*}^2 \} W(\alpha, s, t) \quad , \quad (2.11)$$

$$\dot{\chi}(\xi, s, t) = -\frac{\gamma}{2} \{ (2\bar{n} + 1 - s) \xi \xi^* + \xi^* \partial_{\xi^*} + \xi \partial_\xi \} \chi(\xi, s, t) \quad . \quad (2.12)$$

Using the method of characteristics we will solve for the dynamics of this model for an initial state $\rho = \sum_{\alpha,\beta} N_{\beta\alpha} |\alpha\rangle\langle\beta|$ where $\text{Tr}\rho = \sum_{\alpha\beta} N_{\beta\alpha} \langle\beta|\alpha\rangle = 1$. Thus ρ is a general superposition of coherent states at $t = 0$. (Coherent states are eigenstates of the lowering operator of the oscillator algebra i.e. $a|\alpha\rangle = \alpha|\alpha\rangle$). Since the coherent states are overcomplete any initial state may be represented thus. Let us solve (2.12) for $\chi(\xi, s = -1, t)$ when $\rho(t = 0) = |\alpha_0\rangle\langle\beta_0|$. From (2.5) and using $Q(\alpha) = \langle\alpha|\rho|\alpha\rangle = W(\alpha, -1)$ we get

$$\chi(\xi, -1, t) = \int \frac{d^2\alpha}{\pi} e^{\xi\alpha^* - \xi^*\alpha} Q(\alpha, t) . \quad (2.13)$$

Using this we can calculate the form of the characteristic function χ at $t = 0$ to be

$$\chi(\xi, -1, t = 0) = \langle\beta_0|\alpha_0\rangle \exp[-|\xi|^2 - \xi^*\alpha_0 + \beta_0^*\xi] . \quad (2.14)$$

For $s = -1$ we can rewrite (2.12) as

$$\chi_{,t} + \frac{\gamma}{2}a\chi_{,a} + \frac{\gamma}{2}b\chi_{,b} = -\gamma(1 + \bar{n})ab\chi , \quad (2.15)$$

where $a = \xi$ and $b = \xi^*$ are now treated as independent variables. Looking at the characteristics of (2.15) we see that

$$dt = \frac{da}{\gamma a/2} = \frac{db}{\gamma b/2} = \frac{d\chi}{-\gamma(1 + \bar{n})ab\chi} . \quad (2.16)$$

Integrating this we obtain three constants,

$$k_1 = \frac{2}{\gamma} \ln a - t , \quad k_2 = \frac{2}{\gamma} \ln b - t , \quad (2.17)$$

$$k_3 = -\frac{\ln \chi}{1 + \bar{n}} - ab . \quad (2.18)$$

Solving for a, b and χ in terms of k_i and inserting these into (2.14) at $t = 0$ gives a relationship between the k_i which is valid for $t \geq 0$. Re-inserting the t dependence back into the k_i and going back to the Heisenberg picture yields

$$\chi(\xi, s = -1) = \langle\beta_0|\alpha_0\rangle \exp[-\kappa(t)|\xi|^2 + \beta_0^*(t)\xi - \alpha_0(t)\xi^*] , \quad (2.19)$$

where $\kappa(t) = 1 + \bar{n}(1 - e^{-\gamma t})$, $\beta_0(t) = \beta_0 e^{-(\gamma/2 + i\omega)t}$ and $\alpha_0(t) = \alpha_0 e^{-(\gamma/2 + i\omega)t}$. Inverting (2.13) to obtain the Q distribution we get

$$\begin{aligned} Q(\alpha, t) &= \int \chi(\xi, \xi^*) e^{\alpha\xi^* - \alpha^*\xi} \frac{d^2\xi}{\pi} \\ &= \frac{\langle\beta_0|\alpha_0\rangle}{\kappa(t)} \exp \left[-\frac{1}{\kappa(t)} (\alpha - \beta_0(t))^* (\alpha - \alpha_0(t)) \right] , \end{aligned} \quad (2.20)$$

where we have used (A1) from Appendix A. To obtain the off-diagonal elements $\langle\tilde{\alpha}|\rho(t)|\tilde{\beta}\rangle$ we use the identity

$$\rho = \int \frac{d^2\xi}{\pi} e^{-\xi a^\dagger} e^{\xi^* a} \chi(\xi, \xi^*) , \quad (2.21)$$

and inserting (2.19) we get

$$\begin{aligned} \langle \tilde{\alpha} | \rho(t) | \tilde{\beta} \rangle &= \\ \frac{\langle \beta_0 | \alpha_0 \rangle}{\kappa(t)} \exp \left[-\frac{1}{\kappa(t)} (\tilde{\alpha} - \beta_0(t))^* (\tilde{\beta} - \alpha_0(t)) \right] \langle \tilde{\alpha} | \tilde{\beta} \rangle . \end{aligned} \quad (2.22)$$

Using equation (2.22) we can now examine the general case where $\rho(t=0) = \sum_{\alpha_0\beta_0} N_{\beta_0\alpha_0} |\alpha_0\rangle\langle\beta_0|$ and can evaluate $\rho(t)$ to be

$$\begin{aligned} \rho(t) &= \int \frac{d^2\alpha d^2\beta}{\pi^2} \langle \alpha | \rho | \beta \rangle |\alpha\rangle\langle\beta| , \\ \langle \alpha | \rho(t) | \beta \rangle &= \langle \alpha | e^{\int \mathcal{L} dt} \rho(0) | \beta \rangle , \end{aligned} \quad (2.23)$$

giving

$$\rho(t) = \sum_{\alpha_0\beta_0} \frac{N_{\beta_0\alpha_0}}{\kappa(t)} \langle \beta_0 | \alpha_0 \rangle \int \frac{d^2\alpha d^2\beta}{\pi^2} \langle \alpha | \beta \rangle \exp \left[-\frac{1}{\kappa(t)} (\alpha - \alpha_0(t))^* (\beta - \beta_0(t)) \right] |\alpha\rangle\langle\beta| , \quad (2.24)$$

where \mathcal{L} is the Markov super-operator (2.2). With this the dynamics is completely solved. However, to compare the decay of off-diagonal elements of the reduced density matrix in other bases we will also compute ρ in the position x , momentum p and number basis.

To compute the off-diagonal elements in the position basis we again use equation (2.21), giving

$$\langle x | \rho(t) | y \rangle = \langle x | \int \frac{d^2\xi}{\pi} e^{-\xi a^\dagger} e^{\xi^* a} \chi(\xi, t) | y \rangle , \quad (2.25)$$

where $\chi(\xi, t)$ is given in (2.19). From Appendix B we have

$$\langle x | e^{-\xi a^\dagger} e^{\xi^* a} | y \rangle = \exp \left[\frac{|\xi|^2}{2} + \frac{\xi^{*2} - \xi^2}{4} + x \sqrt{\frac{\omega}{2\hbar}} (\xi^* - \xi) \right] \delta(\xi_x + \sqrt{\frac{\omega}{2\hbar}}(x - y)) \sqrt{\frac{\omega}{2\hbar}} . \quad (2.26)$$

For $\rho(t=0) = \sum_{\alpha_0\beta_0} N_{\beta_0\alpha_0} |\alpha_0\rangle\langle\beta_0|$ we can perform the gaussian integrations in (2.25) and use the result (2.26) to finally get

$$\begin{aligned} \langle x | \rho(t) | y \rangle &= \\ \sqrt{\frac{2}{\pi\sigma_+^2}} \sum_{\alpha_0\beta_0} N_{\beta_0\alpha_0} \langle \beta_0 | \alpha_0 \rangle \times \\ \exp \left[-\frac{1}{2\sigma_-^2} X_-^2 + \frac{1}{\sqrt{\sigma_- \sigma_+}} (\alpha_0(t) - \beta_0^*(t)) X_- - \frac{1}{2\sigma_+^2} (X_+ - \sqrt{\sigma_+ \sigma_-} (\alpha_0(t) + \beta_0^*(t)))^2 \right] , \end{aligned} \quad (2.27)$$

where

$$\sigma_+^2 = \frac{2\hbar}{\omega}(2\kappa(t) - 1) , \quad \sigma_-^2 = \frac{2\hbar}{\omega(2\kappa(t) - 1)} , \quad (2.28)$$

and $X_\pm = x \pm y$. Using similar methods we can calculate the elements in the momentum basis to be

$$\begin{aligned} \langle p_1 | \rho(t) | p_2 \rangle = & \\ \frac{1}{\hbar\omega} \sqrt{\frac{\tilde{\sigma}_-^2}{2\pi}} \sum_{\alpha_0 \beta_0} N_{\beta_0 \alpha_0} \langle \beta_0 | \alpha_0 \rangle & \\ \exp \left[-\frac{1}{2\tilde{\sigma}_-^2} P_-^2 + \frac{i}{\sqrt{\tilde{\sigma}_- \tilde{\sigma}_+}} (\beta_0^*(t) + \alpha_0(t)) P_- - \frac{1}{2\tilde{\sigma}_+^2} (P_+ - i\sqrt{\tilde{\sigma}_- \tilde{\sigma}_+} (\alpha_0(t) - \beta_0^*(t)))^2 \right] , & \end{aligned} \quad (2.29)$$

where

$$\tilde{\sigma}_+^2 = 2\hbar\omega(2\kappa(t) - 1) , \quad \tilde{\sigma}_-^2 = \frac{2\hbar\omega}{2\kappa(t) - 1} , \quad (2.30)$$

and $P_\pm = p_1 \pm p_2$. To calculate the elements of ρ in the number basis is slightly tedious. Noting that

$$\langle n | \rho | m \rangle = \int \frac{d^2\alpha d^2\beta}{\pi^2} \langle n | \alpha \rangle \langle \alpha | \rho | \beta \rangle \langle \beta | m \rangle , \quad \langle n | \alpha \rangle = e^{-|\alpha|^2/2} \alpha^n / \sqrt{n!} , \quad (2.31)$$

and the identity

$$A^n = \int \frac{d^2\alpha}{\pi} \alpha^n e^{-|\alpha|^2 + A\alpha^*} = \int \frac{d^2\alpha}{\pi} \alpha^{*n} e^{-|\alpha|^2 + A\alpha} , \quad (2.32)$$

we find, with some work, that

$$\begin{aligned} \langle n | \rho(t) | m \rangle = & \\ \frac{1}{\kappa(t)} \sum_{\alpha_0 \beta_0} N_{\beta_0 \alpha_0} \langle \beta_0 | \alpha_0 \rangle e^{-\frac{1}{\kappa(t)} \alpha_0^*(t) \beta_0(t)} \Pi^{nm}(\alpha_0, \beta_0, t) \left(\frac{\alpha_0^*(t)}{\kappa(t)} \right)^m \left(\frac{\beta_0(t)}{\kappa(t)} \right)^n , & \end{aligned} \quad (2.33)$$

where

$$\Pi^{nm}(\alpha_0, \beta_0, t) \equiv \sum_{s=0}^{\min(n,m)} \frac{n!m!}{s!(n-s)!(m-s)!} \left(\frac{\kappa(t)(\kappa(t) - 1)e^{\kappa(t)t}}{\beta_0 \alpha_0^*} \right)^s . \quad (2.34)$$

We note that even in the case where $\rho(t=0) = |\alpha_0\rangle\langle\alpha_0|$ equations (2.33) and (2.34) do not reduce to a simple tractable forms. In what follows we will not make much use of equations (2.33) and (2.34) except to note that they become diagonal as $t \rightarrow \infty$. Finally we note that the results for $\langle x | \rho | y \rangle$ found here agree with those found in [24].

III. ENVIRONMENT INDUCED SUPERSELECTION

We now wish to examine the behaviour of the off-diagonal elements of ρ in a number of different bases and to determine whether EIS occurs. There have been a number of proposals for the measure of the degree of diagonality of the density matrix [25,26]. These have mostly been defined however for a gaussian ρ . Here we expand these definitions to a measure Ω_α of the “diagonality” of a general density matrix in a particular basis α (α here denotes a general basis i.e. x, p, n or coherent state).

From recent work [27,28,11,10] there appears to exist some competition between the attainment of a stable predictable pointer basis and the attainment of EIS type decoherence. In this paradigm one must be careful not to confuse the loss of coherence with the loss of interference. In this paper we interpret EIS as a process whereby the *coherence* of the state is lost. From Zurek’s earlier work the signature of decoherence was the vanishing of the *coefficients* of the off-diagonal elements of the reduced density matrix of the system in a given basis. The decay of the interference in a particular basis was due to the decay of these coefficients. However, the loss of interference can also be brought about by sending the overlap of the off-diagonal states in that basis to zero. For example, in Young’s slits, when one sets the slit separation to be very large in comparison to the screen distance, the magnitude of the interference terms decays due to the vanishing of the overlap of these off-diagonal elements in the position basis. (There may, of course, be significant overlap in other bases, eg. the momentum basis.) In this case the loss of interference is *not* due to the decay of the coefficients associated with these off-diagonal elements. The state remains pure, no coherences have been destroyed yet the quasi-probability distribution function *tends* towards (but is never exactly) that displayed by a true probability distribution. Recently decoherence in the EIS paradigm has been defined as the absence of interference fringes in the Wigner function in the special case of a superposition of two gaussians [9]. These interference fringes decay in magnitude for large separations of the two gaussians. They never disappear except in the case of infinite separation. The fringes also vanish dynamically with time if one introduces a coupling with an infinite bath. In the latter case coherence has been lost and the off-diagonal coefficients decay while in the former case the interference terms vanish because the overlap of the off-diagonal elements in the chosen basis goes to zero. Thus, while the loss of coherence necessitates the loss of interference the loss of interference need not result in a loss of coherence. In this paper we follow the spirit of earlier work and define Environment Induced Superselection as the dynamical process whereby the reduced state of the system deviates from a single ray in Hilbert space and moves towards a description as an improper mixture (for more information on improper mixtures see [29]).

The most obvious measure of the deviation of a given state ρ from being a ray in Hilbert space is the statistical distance between the state ρ and the closest mixed state [30–32]

$$D_1(\rho_1) \equiv \min_{\rho_m \in \text{diag } \rho} d_B^2(\rho_1, \rho_m) , \quad (3.1)$$

where

$$d_B^2(\rho_1, \rho_2) = 2(1 - \text{Tr} \sqrt{\rho_1^{1/2} \rho_2 \rho_1^{1/2}}) , \quad (3.2)$$

is the Bures metric on the state space of all pure and impure states ρ [31]. Equation (3.2)

is very difficult to evaluate in general. Instead we will generalize a measure of coherence for gaussian ρ first introduced by Morikawa [25] and used later in [33,26]. For ρ of the form

$$\langle \bar{x} | \rho | x \rangle = D \exp \left[-A(x - \bar{x})^2 - B(x + \bar{x})^2 + iC(x - \bar{x})(x + \bar{x}) \right] , \quad (3.3)$$

Morikawa defined a measure of coherence to be

$$\text{QD} \equiv \frac{A}{B} . \quad (3.4)$$

When $\text{QD} > 1$ the magnitude of the elements of the density matrix in the position basis are peaked preferentially along the diagonal $x = \bar{x}$. For ρ pure, $\text{QD} = 1$. The natural generalization of this measure to non-gaussian ρ is to compute the ratio of the off-diagonal variance to the on-diagonal variance of the squared magnitude of ρ in a specified basis $\hat{\eta}|\eta\rangle = \eta|\eta\rangle$, that is

$$\Omega_\eta \equiv \frac{\int \int d\eta_1 d\eta_2 \langle \eta_1 | \rho | \eta_2 \rangle \langle \eta_2 | \rho | \eta_1 \rangle |\eta_1 - \eta_2|^2}{\int \int d\eta_1 d\eta_2 \langle \eta_1 | \rho | \eta_2 \rangle \langle \eta_2 | \rho | \eta_1 \rangle |\eta_1 + \eta_2 - \langle \eta_1 + \eta_2 \rangle|^2} . \quad (3.5)$$

For ρ gaussian and $|\eta\rangle = |x\rangle$, $\Omega_x = 1/\text{QD}$ from (3.4). For pure states, $\rho = |\psi\rangle\langle\psi|$ and $\Omega_\eta = 1 \ \forall \eta$. This makes sense as no pure state undergoing unitary evolution can lose its coherence. The offset in the denominator gives the variance of $\mathcal{S} \equiv |\langle \eta_1 | \rho | \eta_2 \rangle|^2$ about the mean $\langle \eta \rangle$. Distributions of \mathcal{S} which are concentrated along $\eta_- = \eta_1 - \eta_2 = 0$ result in $\Omega_\eta < 1$. As \mathcal{S} becomes more concentrated along $\eta_- = 0$, Ω_η decreases to zero. From (3.5) the value of Ω_η depends on the basis chosen. This we expect as some basis will display a greater peaking about their diagonal than others. The advantage of the dimensionless measure (3.5) is that one can compare Ω_η and $\dot{\Omega}_\eta$ for various bases $|\eta\rangle$. One can also consider bases which are discrete. In this case the integrals in (3.5) become discrete sums. Heuristically the measure Ω_η tells us how damped the off-diagonal coherences are in the basis $|\eta\rangle$. For pure states, no matter what the configuration, these “off-diagonal” coherences never vanish. For every contribution to the “on-diagonal” elements of ρ there are equal contributions to the “off-diagonal” elements. A classic example of this is the superposition of two localized coherent states $|\psi\rangle = |\alpha\rangle + |-\alpha\rangle$. For a pictorial description of \mathcal{S} for this state see Zurek [3]. In this example the heights of the four peaks in Zurek’s plots do not change with increasing $|\alpha|^2$. No coherence is lost, however the interference diminishes.

A quantification of the amount of interference present in a state is generally given by the magnitude of a typical off-diagonal element. For the special case of two superposed gaussians a measure of the fringe visibility of the ripples of the Wigner function has been proposed [9]. Another quantification of the degree of “off-diagonality” would be

$$\hat{\Omega}_\eta \equiv \frac{1}{\text{Tr } \rho^2} \int \int d\eta_1 d\eta_2 \langle \eta_1 | \rho | \eta_2 \rangle \langle \eta_2 | \rho | \eta_1 \rangle |\eta_1 - \eta_2|^2 . \quad (3.6)$$

This gives a measure of the mean squared variance of the distribution $|\rho(\eta_1, \eta_2)|^2$ along the off-diagonal. Because this measure has dimensions of $|\eta|^2$, comparisons of $\hat{\Omega}$ between bases are made complicated through the different length scales associated with each basis. For a comparison between bases one should use the dimensionless measure (3.5). We also note that while the discrete version of (3.6) vanishes for $\rho \rightarrow \text{diagonal}$ (eg. spin 1/2 particle), for

$$\begin{aligned}\rho &= |\psi\rangle\langle\psi| \ , \\ 2|\psi\rangle &= |\alpha\rangle + |-\alpha\rangle \ ,\end{aligned}\tag{3.7}$$

the continuous version of (3.6) evaluated in the coherent state basis increases like $|\alpha|^2$. Although (3.5) and (3.6) seem to be natural candidates for a measure of “diagonality” in the EIS sense we see that the prototypical state (3.7) is not diagonal nor even approximately diagonal in the coherent state basis. It may be said that since the ripples in the Wigner function decrease in size as $|\alpha|^2 \rightarrow \infty$ one should treat this state in this limit as “interferenceless” or “classical”, This I believe is incorrect. The Wigner function for this state always possesses negative regions for all $|\alpha| \neq \infty$. The interference represented by these would be manifested in the expectation values of observables which have support in these regions. The effects of EIS are to cause these negative regions to disappear in a finite time. However, as pointed out by Bell, positivity of the Wigner function is a necessary but *not* sufficient condition that the Wigner distribution correctly represents a true probability distribution [34]. This connection will be made clearer in [35]. Finally we can use the relation

$$\langle x|\rho|y\rangle = \int dp e^{ip(y-x)/\hbar} W(p, \frac{x+y}{2}) \ ,\tag{3.8}$$

to rewrite (3.6) as

$$\hat{\Omega}_\eta = \frac{1}{\int dp dx W^2(p, x)} \times \left\{ 1 + 8\hbar\omega \int dp dx \left[W(p, x) \left(\frac{d^2}{\omega^2 dx^2} + \frac{d^2}{dp^2} \right) W(p, x) - \left(\frac{d}{\omega dx} W \right)^2 - \left(\frac{d}{dp} W \right)^2 \right] \right\} \ ,\tag{3.9}$$

where we have used

$$\text{Tr } \rho^2 = \int \frac{d^2\alpha}{\pi} W^2(\alpha, \alpha^*) = \int \frac{dp dx}{2\pi\hbar} W^2(p, x) \ .\tag{3.10}$$

In what follows we will use Ω_η , (3.5) as a measure of diagonality. Taking $\rho(t=0) = |\alpha_0\rangle\langle\alpha_0|$ we can compute

$$\Omega_x(t) = \Omega_p(t) = \Omega_\alpha^2 = \frac{\sigma_-^2}{\sigma_+^2} = \frac{1}{(2\kappa(t) - 1)^2} \ .\tag{3.11}$$

Recalling that $\kappa(t) = 1 + \bar{n}(1 - e^{-\gamma t})$, (2.19), this result would indicate that we obtain a greater degree of diagonalisation in the x and p basis than in the coherent state basis α . At $t=0$, since the system begins in a pure state, $\Omega = 1$ for all bases. Thus Environmental Induced Superselection has occurred in all three bases. From (2.34), $|\langle n+m|\rho(t)|n-m\rangle| \sim e^{-m\gamma t}$ and thus ρ also diagonalises in the number basis, however the exact form of Ω_n is quite complicated and will not be given here. For the case of a completely general initial state we only quote the result in the position basis. For $\rho(t=0) = \sum N_{ij}|\alpha_j\rangle\langle\alpha_i|$ one finds

$$\Omega_x = \frac{\sigma_-^2}{\sigma_+^2} \left\{ \frac{\sum_{ijkl} N_{ij} N_{kl} \langle\alpha_i|\alpha_j\rangle\langle\alpha_k|\alpha_l\rangle e^{A_+^2\sigma_+^2 + A_-^2\sigma_-^2 + B} [1 + 2A_-^2\sigma_-^2]}{\sum_{ijkl} N_{ij} N_{kl} \langle\alpha_i|\alpha_j\rangle\langle\alpha_k|\alpha_l\rangle e^{A_+^2\sigma_+^2 + A_-^2\sigma_-^2 + B} [1 + 2A_+^2\sigma_+^2 + 8\langle x\rangle(\frac{\langle x\rangle}{\sigma_+^2} - A_+)]} \right\} \ ,\tag{3.12}$$

where

$$A_- = \frac{-1}{2\sqrt{\sigma_- \sigma_+}} [\alpha_i^*(t) - \alpha_k^*(t) + \alpha_l(t) - \alpha_j(t)] \quad , \quad (3.13)$$

$$A_+ = \frac{\sqrt{\sigma_+ \sigma_-}}{2\sigma_+^2} [\alpha_i^*(t) + \alpha_k^*(t) + \alpha_j(t) + \alpha_l(t)] \quad , \quad (3.14)$$

and

$$B = - [(\alpha_i^*(t) + \alpha_j(t))^2 + (\alpha_k^*(t) + \alpha_l(t))^2] \left(\frac{\sigma_-}{2\sigma_+} \right) \quad . \quad (3.15)$$

A similar result holds for Ω_p where we replace A_\pm with

$$\begin{aligned} A_+ &= \frac{i}{2\sqrt{\sigma_+ \sigma_-}} [\alpha_i^*(t) - \alpha_k^*(t) + \alpha_j(t) - \alpha_l(t)] \quad , \\ A_- &= \frac{-i\sqrt{\sigma_+ \sigma_-}}{2\sigma_+^2} [\alpha_i^*(t) + \alpha_k^*(t) - \alpha_j(t) - \alpha_l(t)] \quad . \end{aligned} \quad (3.16)$$

To obtain a clearer understanding of Ω let us compute Ω_x for the particular state $\rho_2 = \frac{1}{2}|\alpha_0\rangle\langle\alpha_0| + \frac{1}{2}|-\alpha_0\rangle\langle-\alpha_0|$ where $\alpha_0 = (\omega\bar{x} + i\bar{p})/\sqrt{2\hbar\omega}$. Denoting $\rho_1 = |\alpha_0\rangle\langle\alpha_0|$ we find that $\Omega_{2x} = \Omega_{1x}\hat{\Omega}_{2x}$ where $\Omega_{1x} = \sigma_-^2/\sigma_+^2$, (from above) and

$$\hat{\Omega}_{2x} = \left[\frac{1 + \frac{1}{2}(1 - \frac{2\bar{p}^2\sigma_-^2}{\hbar^2})e^{-4\bar{x}^2/\sigma_+^2 - \bar{p}^2\sigma_-^2/\hbar^2}}{1 + \frac{8\bar{x}^2}{\sigma_+^2} + e^{-4\bar{x}^2/\sigma_+^2 - \bar{p}^2\sigma_-^2/\hbar^2}} \right] \quad , \quad (3.17)$$

where σ_\pm are defined in (2.26) and evaluated at $t = 0$. Since $\hat{\Omega}_{2x} \leq 1$ we see that the coherence of a mixture of two Glauber states is less than the coherence of just one. The x -coherence of the mixture ρ_2 decreases inversely as $(\bar{x}/\sigma_+)^2$. Thus packets which are separated in position by amounts significantly greater than their half widths possess lower x -coherence. Also $\hat{\Omega}_{2x}$ depends very weakly on the momentum \bar{p} except for small \bar{x} where the overcompleteness property of the Glauber states cause interference-like behaviour in the x basis. A plot of (3.17) is given in Fig 1.

The dynamical evolution of Ω_{2x} is given again by equation (3.17) with $\sigma_\pm(t)$ given by (2.26) while

$$\bar{x}(t) = e^{-\gamma t/2} \left[\bar{x}(0) \cos \omega t + \frac{\bar{p}(0)}{\omega} \sin \omega t \right] \quad , \quad (3.18)$$

$$\bar{p}(t) = e^{-\gamma t/2} [\bar{p}(0) \cos \omega t - \omega \bar{x}(0) \sin \omega t] \quad . \quad (3.19)$$

$$(3.20)$$

Essentially, the evolution begins at $t = 0$ with the two Glauber states located symmetrically about the origin at positions $\pm\alpha_0 \in \mathbb{C}$. Since the Glauber states are overcomplete there is some overlap between the two. As time progresses, the state, represented loosely as two superposed gaussians in the Wigner distribution, spirals in towards the origin while the half widths of each gaussian spreads with time for nonzero \bar{n} . The system motion complicates

the interpretation and instead, one can return to the interaction picture. In the interaction picture, for a fixed choice of $\alpha_0(t=0)$ where $\bar{x} \gg \sqrt{2\hbar/\omega}$, Ω_{2x} decreases below the value Ω_{1x} due to the large separation in position between the two gaussians. However, as $t \rightarrow \infty$, $\hat{\Omega}_{2x} \rightarrow 1$ and thus the coherence of $\rho_2(t)$ tends to the same value as the coherence of $\rho_1(t)$ in this limit. This is not surprising as the final states are the same. As the bath temperature is raised the time for Ω_{2x} to reach a given value decreases since the gaussian's half widths become larger and thus Ω_{1x} decreases faster. Also, we note that when the \bar{x} separation is large, the state may possess a *lower* coherence than in the equilibrium state at $t = \infty$. This phenomena is also seen in the relaxation behaviour of squeezed states in a thermal bath. The coherence behaviour of extended quantum states and the relation of the Ω measure of coherence to more information theoretic measures will be treated in another paper [35].

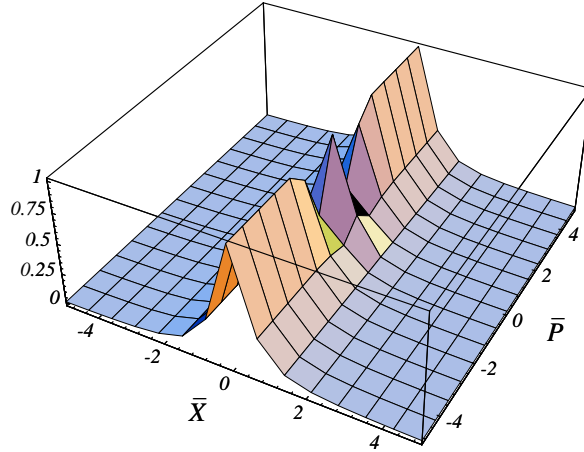


Fig 1. Surface plot of Ω_x for state ρ_2 with $\omega = 1$ and $\hbar = 1$.

To conclude and summarize: we have described the dynamics of the reduced density matrix of a harmonic oscillator linearly coupled to an infinite bath of oscillators in the quantum optical regime. We have expressed the results in the position, momentum, number and coherent state basis. After considering a number of proposals for a measure of the “diagonality” or degree of coherence of the state ρ we have advanced Ω_η (3.5) as a natural measure. We calculated this quantity in the above mentioned bases for an initial Glauber state and found that coherence was lost in each basis. The coherence loss rate for this initial state was found to be identical in the x and p basis while the loss rate in the coherent state basis was smaller. Expressions for the coherence of a general initial state were found in both the x and p bases. Finally, the position coherence for an initial mixture of two coherent states was calculated and was found to be more “diagonal” than that of a single coherent state, especially if the position separation between the two coherent states is large. Although the measure Ω_η may seem to be a “natural” generalization of the early definitions [25] it would be interesting to understand the information theoretic aspects of such measures. Of more interest would be the link between the “diagonality” of ρ and the representation of ρ by a true probability distribution. This work is in progress [35].

IV. CONSISTENT PHASE-SPACE HISTORIES

Consistent Histories is an alternate paradigm developed by Hartle and Gell-Mann [4,36,18,37] which attempts to describe the emergence of the “Quasiclassical” world from the quantum world. The primary ingredients of this paradigm are coarse grained histories and the decoherence functional. Essentially, a coarse grained history is a partial description of the temporal evolution of the total system+environment where one has “coarse grained” away information either by (i) ignoring some of the coordinates for all time (ie. the environment) (ii) only describing the configuration of the privileged coordinates at certain times (iii) projecting the privileged system coordinates at specified time on to quantities of interest eg. binning, averaging etc. The resulting coarse grained histories may be described in some cases (those treated here) by history independent temporally ordered strings of projectors P_i^α , where the P_i^α are a complete and exclusive set of projectors on to a set of alternatives labeled by $\{\alpha\}$ and where P_i^α is the i th projector in the set $P^{\{\alpha\}}$. In the past, calculations have concentrated on sequences on projectors in position using the Caldeira-Leggett model [5,38]. In this section we expand these calculations to projectors on phase-space and consider coarse grained phase-space histories of the privileged system harmonic oscillator in the quantum optical regime. In the Consistent Histories paradigm a probability is assigned to a coarse grained history via the rule

$$P(h) \equiv \text{Tr}[P_h \rho P_h^\dagger] , \quad (4.1)$$

where P_h is the temporally ordered string of projectors corresponding to the coarse grained history h ie.

$$P_h \equiv P_{i_N}^{\alpha_N} \mathcal{U}(\Delta t_N) P_{i_{N-1}}^{\alpha_{N-1}} \mathcal{U}(\Delta t_{N-1}) \cdots P_{i_1}^{\alpha_1} , \quad (4.2)$$

and where \mathcal{U} is the propagator for the full system-environment. We now assume that these projectors factor out the environment and can thus be written in the form $P_i^\alpha = \tilde{P}_{iS}^\alpha \otimes I_E$. In the Markov regime the dynamics of system’s reduced density operator is purely local in time and $\rho_S \equiv \text{Tr}_E \rho$ satisfies partial differential equation which is first order in time and contains only system quantities. In this regime we can take the trace in (4.1) over then environment and arrive at

$$P(h) = \text{Tr}_S \left[\tilde{P}_{i_N}^{\alpha_N} e^{\int_{t_{N-1}}^{t_N} \mathcal{L} dt} \left[\tilde{P}_{i_{N-1}}^{\alpha_{N-1}} \cdots \rho_S \cdots \right] \right] , \quad (4.3)$$

where \tilde{P} acts only on the system’s Hilbert space, ρ_S is the reduced density operator for the system and we have again used (2.2) to evolve the system’s reduced density operator from t_{i-1} to t_i . A similar method is used in [39]. Thus in the Markov regime the effects of the coupling to the environment are taken care of through the superoperator \mathcal{L} and the calculation (4.1) reduces to a calculation on the system only.

At the heart of this formalism is the requirement that these probabilities satisfy the sum rule

$$P(h) = \sum_{\tilde{h} \in h} P(\tilde{h}) , \quad (4.4)$$

where \tilde{h} is a finer graining of h . For the sum rule (4.4) to hold with the graining \tilde{h} we must have

$$\text{Re } D[\tilde{h}', \tilde{h}] = 0 \quad \forall \tilde{h}', \tilde{h} \in h, \tilde{h}' \neq \tilde{h} , \quad (4.5)$$

where

$$D[\tilde{h}', \tilde{h}] = \text{Tr}[P_{\tilde{h}} \rho P_{\tilde{h}'}^\dagger] , \quad (4.6)$$

is known as the Decoherence Functional [36]. Condition (4.6) is often termed *Weak Consistency*. A stronger, although sometimes argued more physical, condition is

$$D[\tilde{h}', \tilde{h}] = 0 \quad \forall \tilde{h}', \tilde{h} \in h, \tilde{h}' \neq \tilde{h} , \quad (4.7)$$

and is called *Medium Consistency*. A major goal in this approach is the identification of all those sets of histories which are exactly consistent and from this, an identification of those histories which yield a “Quasiclassical” world. We do not wish to give an in-depth review of this formalism and refer the reader the excellent articles of Gell-Mann and Hartle [4,36,18,37].

It has been recognized that sets of histories which are *exactly* consistent may be characterized by very exotic and un-physical system variables [27]. To examine the consistency properties of histories parametrised by (perhaps) more interesting and physical variables (ie. position and momentum) a measure of the degree of consistency of a set of histories $\{\tilde{h}\}$ has been advanced by Dowker and Halliwell [5]

$$\epsilon \equiv \max_{\substack{\tilde{h}', \tilde{h} \in \{h\} \\ \tilde{h}' \neq \tilde{h}}} \frac{\text{Re } D[\tilde{h}', \tilde{h}]}{\sqrt{D[\tilde{h}, \tilde{h}] D[\tilde{h}', \tilde{h}']}} . \quad (4.8)$$

As $\{\tilde{h}\}$ becomes more consistent ϵ decreases. Dowker and Halliwell have also argued that in general ϵ decreases as the graining size is increased. Only for very artificial grainings, where the phases of the constituent histories in the coarser graining add constructively, does ϵ remain unchanged or actually increase. This point will be important later. As mentioned above, all previous calculations have looked at histories on position space. However we normally consider classical dynamics as operating within phase-space. It would be of great interest to see if phase-space histories of the system become more consistent with the addition of an interaction with an external bath. In the previous section on Environmental Induced Superselection we saw that the quantum optical model gave EIS in both the x and p bases. Here we wish to see whether phase-space histories for the privileged harmonic oscillator displays consistency and if so, to what degree? How does the degree of consistency depend on the model parameters ie. γ , \bar{n} graining etc? In the following we will consider two-time projected histories on the phase-space of the privileged oscillator. We will again couple the system to an external bath and use the solutions of the master equation (2.2) found in section II to evolve ρ_S between projections. We will calculate the Decoherence Functional and the degree ϵ of consistency of these histories. This ϵ will be a function of the equilibrium number of photons per mode in the bath ($\bar{n} = 1/(\exp \frac{\hbar\omega}{kT} - 1)$), the time t , the initial state of the system $\rho(t=0)$ and the phase-space projectors chosen (ie. the graining). We will

choose gaussian type “projectors” on phase-space and will find that ϵ decreases with time t , increasing bath temperature T and final grain size σ_2 but anomalously decreases with initial grain size σ_1 . This anomalous behaviour can also be seen in other calculations of Decoherence Functionals for similar models [5,38]. We conclude that the introduction of the bath causes the phase-space histories to become more consistent and less peaked about the classical path. Finally we note that Consistent Histories calculations tend to give rather lengthy and hard to interpret results. We have found that working with this model has resulted in a more transparent description.

Two-Time Phase Space Histories

We wish to calculate the decoherence functional for two-time projected histories on phase-space, that is

$$\begin{aligned} D[\tilde{h}, \tilde{h}'] &= D[\alpha_3, \alpha_2, \alpha_1, \sigma_2, \sigma_1, t] \\ &= \text{Tr} \left[P(\alpha_3, \sigma_2) e^{\int_{t_1}^{t_2} \mathcal{L} dt} \left[P(\alpha_2, \sigma_1) \rho(t_1) P^\dagger(\alpha_1, \sigma_1) \right] \right] , \end{aligned} \quad (4.9)$$

where the coarse grained history is defined by a projector $P(\alpha, \sigma)$ on the phase-space of the system oscillator at $\alpha \in \mathbb{C}$ of “width” σ . The trace in (4.9) is over the system’s Hilbert space. We will assume, as in [5,38], that P is a *quasi-projector*. In particular we will take P to be gaussian with

$$P(\alpha, \sigma) = \frac{1}{\sigma^2} \int \frac{d^2\beta}{\pi} e^{-\frac{|\beta-\alpha|^2}{\sigma^2}} |\beta\rangle\langle\beta| , \quad (4.10)$$

where $|\beta\rangle$ is a Glauber coherent state. These projectors are complete ie. $\int d^2\alpha/\pi P(\alpha, \sigma) = 1$, but are not exclusive

$$P(\alpha_1, \sigma) P(\alpha_2, \sigma) \neq \delta(\alpha_1 - \alpha_2) P(\alpha_2, \sigma) . \quad (4.11)$$

Using such gaussian “quasi-projectors” makes the calculation analytically tractable. In fact, since true projectors on phase-space do not exist [40] the best one can do is to use such complete but nonexclusive “quasi-projectors”. Indeed, these types of “quasi-projections” play a significant role in the more rigorous “Effects and Operations” theory of quantum measurement [14]. To create other quasi-projectors which are more localized in phase-space one could use instead the marginally overcomplete set of coherent states $\{|\alpha\rangle\}_{VN}$ where one places an ordinary Glauber coherent state $|\alpha\rangle$ at each point of a von Neumann lattice in phase space [41,42].

We shall calculate (4.9) for an arbitrary $\rho(t_1)$ by decomposing $\rho(t_1)$ into coherent states and using (2.24) to evolve to t_2 . We will then consider a $\rho(t_1)$ resulting from the propagation from a time $t_0 < t_1$ of $\rho(t_0)$ which we shall choose to be $\rho(t_0) = |\alpha_0\rangle\langle\alpha_0|$. We will obtain the decoherence functional D and from this calculate the degree of consistency ϵ and peaking \mathcal{P} about the classical trajectory. We will also calculate Ω_h , a measure similar to (3.5), which measures the coherence in D when considered as a density matrix. We find that ϵ and Ω_h behave similarly with respect to their dependence on the parameters in the model.

To begin, we take $\rho(t_1) = \sum_{ij} N_{ij} |\alpha_j\rangle\langle\alpha_i|$ where we have normalized according to $\sum_{ij} N_{ij} \langle\alpha_j|\alpha_i\rangle = 1$. Using (A1),

$$\int \frac{d^2\alpha}{\pi} e^{-A|\alpha|^2 + B\alpha^* + C\alpha} = \frac{1}{A} e^{\frac{BC}{A}} , \quad (4.12)$$

and the definition of a coherent state $|\alpha\rangle = e^{\alpha a^\dagger - \alpha^* a} |0\rangle$ we can compute

$$P(\alpha_2, \sigma_1) \rho(t_1) P^\dagger(\alpha_1, \sigma_1) = \frac{1}{s_1^2} \sum_{ij} \mathbb{F}(\alpha_1, \alpha_2, \alpha_i, \alpha_j, \sigma_1) |\zeta_j\rangle\langle\eta_i| , \quad (4.13)$$

where

$$s_1 = 1 + \sigma_1^2 , \quad s_1 \zeta_j = \sigma_1^2 \alpha_j + \alpha_2 , \quad s_1 \eta_i = \sigma_1^2 \alpha_i + \alpha_1 , \quad (4.14)$$

and

$$\mathbb{F}(\alpha_1, \alpha_2, \alpha_i, \alpha_j, \sigma_1) = \exp \left\{ -\frac{1}{2s_1^2} [(|\alpha_i|^2 + |\alpha_j|^2 + |\alpha_1|^2 + |\alpha_2|^2)(1 + 2\sigma_1^2) + (\alpha_2^* \alpha_j + \alpha_1 \alpha_i^*)(2 + 3\sigma_1^2) + \sigma_1^2(\alpha_2 \alpha_j^* + \alpha_1^* \alpha_j)] \right\} . \quad (4.15)$$

Using (2.24) we can evolve (4.13) from t_1 to $t_2 = t_1 + \Delta t_2$ and denoting this by $\rho_{eff}(t_2)$ we get

$$\rho_{eff}(t_2) = \int \frac{d^2\alpha d^2\beta}{\pi^2} \langle\alpha|\rho_{eff}(t_2)|\beta\rangle |\alpha\rangle\langle\beta| , \quad (4.16)$$

where

$$\langle\alpha|\rho_{eff}(t_2)|\beta\rangle = \frac{1}{s_1^2} \sum_{ij} \frac{\mathbb{F}(\alpha_1, \alpha_2, \alpha_i, \alpha_j, \sigma_1)}{\kappa(\Delta t_2)} \exp \left\{ -\frac{1}{\kappa(\Delta t_2)} (\alpha - \eta_i(\Delta t_2))^* (\beta - \zeta_j(\Delta t_2)) \right\} \langle\alpha|\beta\rangle \langle\eta_i|\zeta_j\rangle , \quad (4.17)$$

where $\zeta_j(t) \equiv e^{-(\gamma/2 + i\omega)t} \zeta_j$ and $\eta_i(t) \equiv e^{-(\gamma + i\omega)t} \eta_i$. The final decoherence functional is thus

$$D(\alpha_3, \alpha_2, \alpha_1, \sigma_2, \sigma_1, t_2) = \frac{1}{\sigma_2^2} \int \frac{d^2\gamma}{\pi} e^{-\frac{|\alpha_3 - \gamma|^2}{\sigma_2^2}} \langle\gamma|\rho_{eff}(t_2)|\gamma\rangle , \quad (4.18)$$

and is thus a gaussian smearing of the Q function of $\rho_{eff}(t_2)$. Note that although Q functions are strictly positive for true density matrices, when $\alpha_1 \neq \alpha_2$, $\rho_{eff}(t_2)$ is *not* a true density matrix and thus $D[\alpha_3, \alpha_2, \alpha_1 \neq \alpha_2]$ may be complex. To calculate (4.18) is quite tedious but one finally arrives at

$$D = \sum_{ij} \frac{N_{ij}}{\sigma_1^2} \mathbb{F}(\alpha_2, \alpha_1, \alpha_i, \alpha_j, \sigma_1) \frac{\langle\eta_i|\zeta_j\rangle}{s_2} \times \exp \left\{ -\frac{|\alpha_3|^2}{\sigma_2} - \frac{\eta_i^*(\Delta t_2) \zeta_j(\Delta t_2)}{\kappa_2} + \frac{(\kappa_2 \alpha_3 + \sigma_2^2 \zeta_j(\Delta t_2))(\kappa_2 \alpha_3 + \sigma_2^2 \eta_i(\Delta t_2))^*}{s_2 \sigma_2^2 \kappa_2} \right\} , \quad (4.19)$$

where $\kappa_2 \equiv \kappa(\Delta t_2)$ and $s_2 \equiv \kappa(\Delta t_2) + \sigma_2^2$. For simplicity we will now turn to the interaction picture and consider our projectors to be stationary in this picture ie. $\alpha_1 \rightarrow \alpha_1 e^{-i\omega t}$. With this, all ω dependence drops out of (4.19). With a bit of rearranging (4.19) can be written in the more convenient form

$$D = \sum_{ij} \frac{N_{ij}}{s_1^2 s_2} \exp \left\{ -\frac{|\alpha_i|^2}{2} - \frac{|\alpha_j|^2}{2} + \frac{(s_1 - 1)^2}{s_1^2} \alpha_i^* \alpha_j G + \alpha_i^* \mathbb{E}(\alpha_1, \alpha_2, \alpha_3) + \alpha_j \mathbb{E}^*(\alpha_2, \alpha_1, \alpha_3) \right. \\ \left. - \frac{|\alpha_3|^2}{s_2} - \frac{|\alpha_1|^2}{s_1} - \frac{|\alpha_2|^2}{s_1} + \frac{\alpha_1^* \alpha_2}{s_1^2} G + (\alpha_3^* \alpha_2 + \alpha_3 \alpha_1^*) \frac{e^{-\gamma \Delta t/2}}{s_1 s_2} \right\} , \quad (4.20)$$

where $G \equiv 1 - e^{-\gamma \Delta t_2}/s_2$ and

$$\mathbb{E}(\theta_1, \theta_2, \theta_3) \equiv \frac{\theta_1}{s_1} + \frac{\sigma_1^2 \theta_2 G}{s_1^2} + \frac{\sigma_1^2 \theta_3 e^{-\gamma \Delta t_2/2}}{s_1 s_2} . \quad (4.21)$$

We are now ready to choose $\rho(t_0) = |\alpha_0\rangle\langle\alpha_0|$. From (2.2) we have

$$\rho(t_1) = \int \frac{d^2 \alpha_j d^2 \alpha_i}{\pi^2} \langle \alpha_j | e^{\int_{t_0}^{t_1} \mathcal{L} dt} \rho(t_0) | \alpha_i \rangle | \alpha_j \rangle \langle \alpha_i | , \quad (4.22)$$

and using (2.24) yields

$$\rho(t_1) = \int \frac{d^2 \alpha_j d^2 \alpha_i}{\kappa(\Delta t_1) \pi^2} e^{-\frac{1}{\kappa(\Delta t_1)} (\alpha_j - \alpha_0 e^{-\gamma \Delta t_1/2})^* (\alpha_i - \alpha_0 e^{-\gamma \Delta t_1/2})} \langle \alpha_j | \alpha_i \rangle | \alpha_j \rangle \langle \alpha_i | , \quad (4.23)$$

where $\Delta t_1 = t_1 - t_0$. Since the action of the decoherence functional is linear we can use (4.20) to obtain the action on $|\alpha_j\rangle\langle\alpha_i|$ and can then perform the integrations in (4.23) to compute the complete decoherence functional. We finally get

$$D = \frac{1}{s_2 \Upsilon} \exp \{ -A |\Delta_1|^2 - B |\Delta_2|^2 + C (\Delta_1^* \Delta_2 + \Delta_1 \Delta_2^*) - D |z|^2 - i \text{Im} \mathbb{A} \} , \quad (4.24)$$

where the definitions of the various quantities appearing in (4.24) are

$$\begin{aligned} \Upsilon &= s_1^2 \kappa_1 - \Phi (s_1 - 1)^2 G , & 4\Upsilon A &= 2s_1 - 1 + \frac{1}{s_2} e^{-\gamma \Delta t_2} (1 - \Phi) , \\ \Phi &= \bar{n} (1 - e^{-\gamma \Delta t_1}) , & s_2 \Upsilon B &= s_1^2 - \Phi , \\ \Delta_1^* &= \alpha_1 + \alpha_2 - 2\alpha_0 e^{-\gamma \Delta t_1/2} , & 2s_2 \Upsilon C &= (s_1 - \Phi) e^{-\gamma \Delta t_2/2} , \\ \Delta_2^* &= \alpha_3 - \alpha_0 e^{-\gamma (\Delta t_2 + \Delta t_1)/2} , & 4\Upsilon D &= 1 + 2s_1 - \frac{1}{s_2} e^{-\gamma \Delta t_2} + \Phi \left[4 - e^{-\gamma \Delta t_2} \frac{(3 - 2s_1)}{s_2} \right] , \\ z &= \alpha_1 - \alpha_2 , & 2\Upsilon \mathbb{A} &= z \left\{ \left[1 - \frac{1}{s_2} e^{-\gamma \Delta t_2} (1 + \Phi) \right] \Delta_1^* + \frac{2}{s_2} [s_1 + \Phi] e^{-\gamma \Delta t_2/2} \Delta_2^* \right\} , \\ G &= 1 - \frac{e^{\gamma \Delta t_2}}{s_2} , & & \\ s_1 &= 1 + \sigma_1^2 , & & \\ s_2 &= \sigma_2^2 + \kappa_2 , & & \\ \kappa_2 &= 1 + \bar{n} (1 - e^{-\gamma \Delta t_2}) , & & \\ \kappa_1 &= 1 + \bar{n} (1 - e^{-\gamma \Delta t_1}) . & & \end{aligned} \quad (4.25)$$

Thus Δ_1 and Δ_2 are the deviations of the projector positions at times t_1 and t_2 from the classical path of the damped coherent state $|\alpha_0\rangle$. The diagonal elements of D , ie. $z = 0$,

are peaked along this classical path ($\Delta_1 = \Delta_2 = 0$). We also note that because of the nonexclusivity of these quasi-projectors the diagonal elements of the decoherence functional do not sum exactly to one. For a decoherence functional of the form (4.24) it is easy to show that the degree ϵ of consistency defined in (4.8) is

$$\epsilon = \max_{\alpha_3 \in \mathbb{C}} e^{(C-D)|z|^2} \cos \mathbb{A} . \quad (4.26)$$

Thus the maximum departure from perfect consistency between a pair of histories occurs for (i) those histories for which α_3 is such that $|\cos \mathbb{A}| = 1$ (ii) for pairs of histories h, h' where the quasi-projectors $P(\alpha_1, \sigma_1)$ and $P(\alpha_2, \sigma_1)$ are separated by their full width at half maximum (FWHM) i.e. $z = 1 + \sigma_1^2 = s_1$. From (i), (ii) and (4.26) we get

$$\begin{aligned} \epsilon &= \exp \left\{ -\frac{s_1}{2\Upsilon} \left[1 - \frac{1}{s_2} e^{-\gamma \Delta t_2} + \Phi \left(2 + e^{-\gamma \Delta t_2} \frac{(s_1 - 1)}{s_2} \right) \right] \right\} \\ &\equiv \exp \{ -\bar{\epsilon}(\sigma_1, \sigma_2, \gamma, \Delta t_2, \Delta t_1, \bar{n}) \} . \end{aligned} \quad (4.27)$$

For $\bar{\epsilon} \gg 1$ we achieve good consistency. To examine the degree of peaking about the classical path we follow Dowker and Halliwell [5] and look at the product of the determinant of the coefficients of Δ_i with the FWHM of the initial and final projectors, that is $\mathcal{P} \sim (AB - C^2)s_1(1 + \sigma_2^2)$. We get

$$\mathcal{P} = \frac{s_1(1 + \sigma_2^2)}{4s_2\Upsilon^2} \left[(s_1^2 - \Phi)(2s_1 - 1) - \Phi e^{-\gamma \Delta t_2} \frac{(s_1 - 1)^2}{s_2} \right] . \quad (4.28)$$

For $\mathcal{P} \gg 1$ we get significant peaking about the classical path. Equations (4.24, 4.25, 4.27, 4.28) are the main results of this section.

Let us examine the short and long time behaviour of $\bar{\epsilon}$ and \mathcal{P} . For $t_2 \sim t_1 \sim 0$ we have

$$\bar{\epsilon} \approx \frac{1}{2s_1} \left[1 - \frac{1}{s_2} \right] . \quad (4.29)$$

Thus as the final graining size σ_2 increases the degree of consistency increases. This is in agreement with the general arguments of Dowker and Halliwell. However, as the initial graining size s_1 is increased, the degree of consistency drops. This anomalous behaviour is also seen in other calculations [5,38]. In the long time limit, with $\Delta t_2 = \Delta t_1 = \tau$ and letting $\tau \rightarrow \infty$ we have

$$\bar{\epsilon} \approx \frac{(2\bar{n} + 1)s_1}{2[s_1^2 + \bar{n}(2s_1 - 1)]} , \quad (4.30)$$

which again decreases with increasing s_1 . However, the dependence on bath temperature, through \bar{n} , is now apparent and thus the consistency increases with increasing T . In the short time limit the peaking is

$$\mathcal{P} \approx \frac{2s_1 - 1}{4s_1} , \quad (4.31)$$

while for long times

$$\mathcal{P} \approx \frac{s_1(s_1^2 - \bar{n})(2s_1 - 1)}{2[s_1^2 + \bar{n}(2s_1 - 1)]^2} \left(\frac{1 + \sigma_2^2}{1 + \bar{n} + \sigma_2^2} \right) . \quad (4.32)$$

The short time and long time limits of \mathcal{P} display the expected behaviour with the peaking getting better with increasing graining size and decreasing bath temperature and time. Plots of the long time behaviors of $\bar{\epsilon}$ and \mathcal{P} are shown in Fig 2.

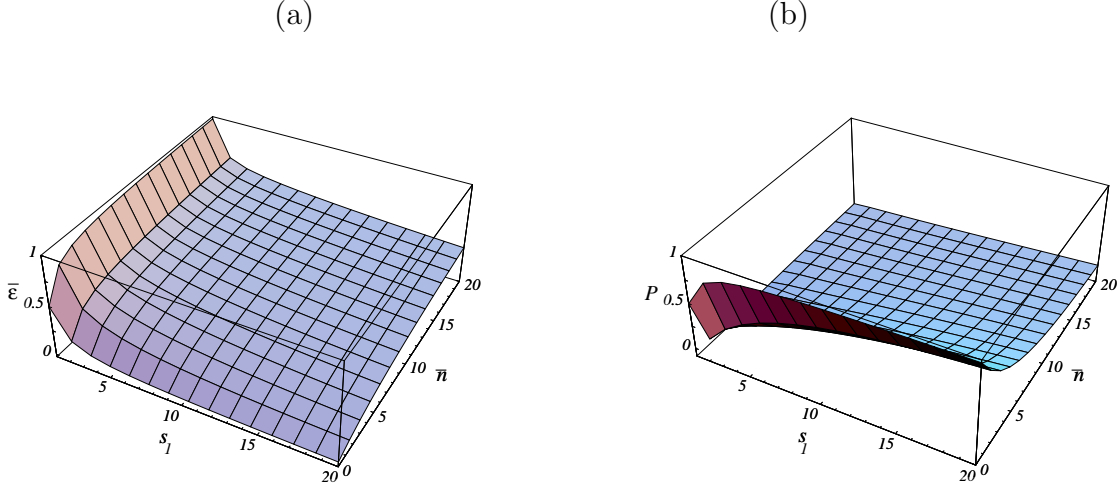


Fig 2. Dependencies of the degree of consistency and degree of peaking in the long time limit on the bath temperature (through \bar{n}) and initial grain size s_1 . Graph (a) plots (4.30) as a function of s_1 and \bar{n} , while graph (b) plots (4.32). We see that increasing the bath temperature (which increases \bar{n}) increases consistency while decreasing peaking. Increasing the initial grain size s_1 *decreases* consistency and increases peaking

We can also examine the degree of consistency by considering the decoherence functional as a density matrix in the space of the coarse grained histories $\{h\}$. With this viewpoint we can adopt the definition (3.5) to give a measure of the concentration of $D[h, h']$ about the diagonal $h = h'$. For a decoherence functional of the form

$$D \sim N \exp \left[-A|\Delta_1|^2 - B|\Delta_2|^2 + C(\Delta_1^* \Delta_2 + \Delta_1 \Delta_2^*) - D|z|^2 - i \text{Im } z \Delta \right] , \quad (4.33)$$

we can calculate

$$\Omega_h = \max_{\Delta_2 \in \mathbb{C}} \frac{\int d^2 \Delta_1 d^2 z D[\Delta_2, \Delta_1, z] D[\Delta_2, \Delta_1, -z] |z|^2}{\int d^2 \Delta_1 d^2 z D[\Delta_2, \Delta_1, z] D[\Delta_2, \Delta_1, -z] |\Delta_1 - \langle \Delta_1 \rangle|^2} . \quad (4.34)$$

We can motivate this measure by noting that for each value of Δ_2 , $D[\Delta_2, \Delta_1, z]$ is very like a density matrix with the on-diagonals labeled by Δ_1 and the off-diagonals labeled by z . Computing (4.34) for a given Δ_2 gives a measure of the degree of coherence between all the possible pairs of histories which have $P(\alpha_3, \sigma_2)$ as their final quasi-projector. The maximum of (4.34) over Δ_2 gives the largest violation of consistency for the complete set of histories labeled by both Δ_2 and Δ_1 . Before computing (4.34) we first obtain $\langle \Delta_1 \rangle$ for a fixed $\Delta_2 \neq 0$ to be

$$\begin{aligned}
\langle \Delta_1 \rangle &= \int \frac{d^2 \Delta_1}{2\pi} N \exp \left[-A|\Delta_1|^2 - B|\Delta_2|^2 + C(\Delta_1^* \Delta_2 + \Delta_1 \Delta_2^*) \right] \Delta_1 \\
&= \frac{NC\Delta_2}{A^2} e^{-(AB-C^2)|\Delta_2|^2/A} ,
\end{aligned} \tag{4.35}$$

which gives $\langle \Delta_1 \rangle = 0$ when $\Delta_2 = 0$ as expected.

After some calculation one obtains the maximum violation of consistency to simply be

$$\begin{aligned}
\Omega_h &= \frac{A}{D} , \\
&= \frac{\left[2s_1 - 1 + \frac{1}{s_2} e^{-\gamma \Delta t_2} (1 - \Upsilon) \right]}{\left[1 + 2s_1 - \frac{1}{s_2} e^{-\gamma \Delta t_2} + \Upsilon \left(4 - \frac{1}{s_2} [3 - 2s_1] e^{-\gamma \Delta t_2} \right) \right]} .
\end{aligned} \tag{4.36}$$

In the short and long time limits we get

$$\Omega_h(t_1 \sim t_2 \rightarrow 0) = \frac{2s_1 - (1 - \frac{1}{s_2})}{2s_1 + (1 - \frac{1}{2s})} \leq 1 , \tag{4.37}$$

$$\Omega_h(t_1 \sim t_2 \rightarrow \infty) = \frac{2s_1 - 1}{2s_1 + 1 + 4\bar{n}} \leq \Omega_h(t_1 \sim t_2 \rightarrow 0) \leq 1 . \tag{4.38}$$

From (4.38) we see that even when $\bar{n} = 0$ the coherence decays. This is due to the vacuum fluctuations. It is interesting to note that all dependence on s_2 drops out of these expressions in the long time limit. Plots of Ω_h for $\Delta t_1 = \Delta t_2 = \tau$ (4.36), are shown in Fig 3. Very similar dependence on the model parameters is shown between Ω_h and $\bar{\epsilon}$, in particular the anomalous s_1 behaviour.

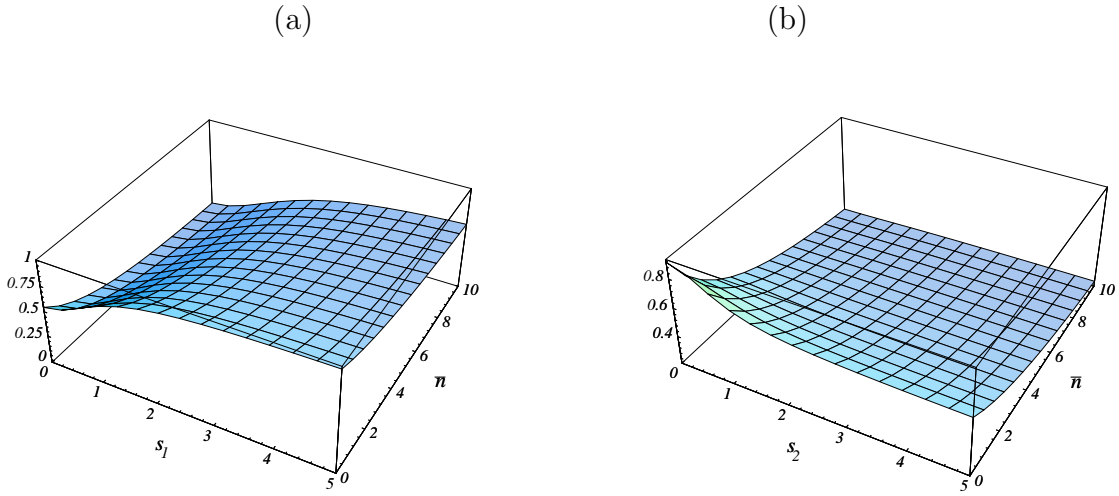


Fig 3. Plots of Ω_h , (4.36) as a function of \bar{n} , s_1 and s_2 . We have set $\gamma = 1$ and $\Delta t_1 = \Delta t_2 = \tau = 0.2$. Graph (a) shows that consistency decreases with increasing initial grain size s_1 .

To conclude and summarize: we have examined the Consistent Histories framework within a quantum optical model using two-time projected phase-space histories. In the Markovian regime the bath can be traced away and its effect completely absorbed into the non-unitary evolution super-operator \mathcal{L} . This is used to evolve the reduced density matrix of the system between projections. Since no true phase-space projectors exist we use “quasi-projectors” which are gaussian convolutions of coherent states. Although these quasi-projectors are complete, they are not totally exclusive and therefore do not exactly meet the specifications of this paradigm. They, however, do provide a tractable model and have been used successfully in other calculations. Moreover, such quasi-projectors arise naturally in more rigorous theories of quantum measurement. We first calculated the “action” of the decoherence functional for these coarse grained histories on the state $|\alpha_j\rangle\langle\alpha_i|$ at t_1 . With this information, the action of the decoherence functional on any state at t_1 can be evaluated through a decomposition of $\rho(t_1)$ into coherent states. This was done for the particular case of $\rho(t_0) = |\alpha_0\rangle\langle\alpha_0|$ where $t_0 \leq t_1$. The resulting decoherence functional was exponential in form and peaked on the classical path of the damped oscillator. Using the measures of Dowker and Halliwell the degree of consistency and the degree of peaking about the classical path were calculated and the short and long time limits found. The behaviour of these quantities were mostly as expected: the consistency gets better with time, increasing bath temperature and final grain size while the peaking is better with increased final and initial grain size and small bath temperature. The consistency *decreased* with increased initial grain size. We also calculated Ω_h , a measure similar to the one proposed in the EIS section of this paper, which quantifies the amount of coherence in the decoherence functional when the decoherence functional is considered as a density matrix on the space of coarse grained histories. This measure displayed similar behaviour to the measure of consistency as advanced by Dowker and Halliwell.

V. CONCLUSION

Upon comparison we see that both EIS and consistency among phase-space histories improves as the bath temperature increases. Altering the coupling strength γ , changes only the time derivatives of the degree of EIS or consistency achieved and not their actual values. A time for “decoherence” may be set equal to the time needed for Ω_η to reach some predetermined value ie. $\Omega_\eta(t_d) = .5$, say. The anomalous dependence of $\bar{\epsilon}$ on s_1 occurs in all the two-time projected history calculations to date. For this to occur in one model for a special type of graining might be understandable within the arguments given by Dowker and Halliwell concerning the constructive interference of the constituent histories in a very special coarse-graining. Since the behaviour is seen in three different calculations with two different models [5,38] one suspects that there is some feature common to all that is upsetting the expected behaviour of increased consistency with increased s_1 . An approximation common to all of these calculations is the use of “quasi-projectors”. It may be that the non-exclusivity of these projectors is somehow overcounting the contributions of “off-diagonal” histories in the decoherence functional. It is thus necessary to resort to true projectors. Since there do not exist true projectors on phase-space one must consider true projectors on to other spaces. An obvious choice is either the position, momentum or number basis. Choosing the latter is the most computationally efficient and was treated in [43]. The results were very

surprising and highlighted the differences between the two paradigms, EIS and CH.

Essentially the goal of EIS is the damping of the coherences present in the system's quantum state. The goal of CH, however, is the attainment of histories whose probabilities satisfy the sum rule. For this to occur the quantum *interferences* between the individual histories must vanish. The damping of quantum coherence and vanishing of quantum interferences need not be synonymous. The decay of coherence necessitates the vanishing of interference but not visa versa. This is clearly shown in the double slit examples of Gell-Mann and Hartle. When the slit separation is much greater than the slit/screen separation the resulting coarse-grained histories are very consistent. However, if the separate beams are then later on recombined somehow, they will display interference. One then says that the histories which include this recombination are no longer consistent. Thus, histories which may begin very consistent, may, later on, become very inconsistent and visa versa. This is so, even though the evolution can be strictly unitary! From the viewpoint of EIS, the coherence of the state is not lost under unitary evolution and thus EIS does not occur. However, if one now inserts the Young's slits in a bath of electrons [36], the resulting histories are even more consistent and furthermore they cannot usually be made less consistent through recombination etc. This is so because the reduced state of the light beams has lost some of it's coherence to the bath of electrons. Thus, both CH and EIS has occurred in the reduced state of the light beams. Since the measure Ω_η was introduced in section III as a measure of coherence, its value as a measure of consistency (which involves the vanishing of interferences) is slightly suspect. However, we would argue that the vanishing of interference between *all* pairs of possible histories in a set of very many alternatives necessitates the vanishing of interference between neighboring histories. For this to happen, as in the Young Slit case for small slit separation, one must have a loss of coherence (usually by introducing a bath).

Finally, it would be of greater interest to have a more information theoretic formulation of the concept of "diagonalisation" and consistency. The introduction of a bath causes the reduced quantum state to lose it's non-local correlations preferentially in certain bases. To link this concept with that of a pointer basis - that basis least effected through the bath interaction - would be of interest. Also to see how the decohered state (either via EIS or CH) satisfies the Bell inequalities would give, perhaps, a more physical measure of what "diagonality" really means. This work is in progress [35]. Also, the quantification of classical correlations via information theory in the CH approach is also important [44].

In this paper we have shown that in this relatively simple model of a quantum open system EIS and CH seem to behave similarly, becoming better with increasing bath temperature and time. The dependence of the consistency on graining also behaves almost as expected. This example shows how the two paradigms are similar. Other calculations also demonstrate the differences between the two [43,26]. Clearly, if both paradigms are to describe an ontology it is essential to discover situations where they differ and to experimentally determine which is correct.

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APPENDIX A: INTEGRAL RELATIONS FOR COHERENT STATES

In this appendix we compile a number of integral results useful for calculations involving coherent states. We refer the reader to [45] for a very complete review of coherent states and their properties. Here we list a number of results.

$$\int \frac{d^2\alpha}{\pi} \exp \{ -A|\alpha|^2 + B\alpha^2 + C\alpha^{*2} + D\alpha + E\alpha^* \} = \frac{1}{\sqrt{A^2 - 4BC}} \exp \left\{ \frac{DEA + E^2B + D^2C}{A^2 - 4BC} \right\} \quad \left\{ \begin{array}{l} A - B - C > 0 \\ A^2 - 4BC > 0 \end{array} \right. , \quad (\text{A1})$$

$$\int \frac{d^2\alpha}{\pi} \{ \alpha, \alpha^* \} e^{-A|\alpha|^2 + B\alpha^* + C\alpha} = \frac{\{B, C\}}{A^2} e^{BC/A} , \quad (\text{A2})$$

$$\int \frac{d^2\alpha}{\pi} \{ \alpha^2, \alpha^{*2} \} e^{-A|\alpha|^2 + B\alpha^* + C\alpha} = \frac{\{B^2, C^2\}}{A^3} e^{BC/A} , \quad (\text{A3})$$

$$\int \frac{d^2\alpha}{\pi} |\alpha|^2 e^{-A|\alpha|^2 + B\alpha^* + C\alpha} = \frac{1}{A^2} \left[1 + \frac{BC}{A} \right] e^{BC/A} , \quad (\text{A4})$$

$$\int \frac{d^2\alpha}{\pi} |\alpha|^4 e^{-A|\alpha|^2 + B\alpha^* + C\alpha} = \frac{1}{A^5} [2A^2 + 4ABC + B^2C^2] e^{BC/A} . \quad (\text{A5})$$

$$(\text{A6})$$

$$\int \frac{d^2\alpha_1 d^2\alpha_2}{\pi} \exp \{ -A_1|\alpha_1|^2 + \alpha_1^*[B_1 + C_1\alpha_2] + D_1\alpha_1 + E_1\alpha_1^{*2} + F_1\alpha_1^2 - A_2|\alpha_2|^2 + C_2\alpha_2^* + D_2\alpha_2 + E_2\alpha_2^{*2} + F_2\alpha_2^2 \} = \frac{1}{\sqrt{\eta}} \exp(\Lambda/\eta) , \quad (\text{A7})$$

where

$$\eta = (A_1^2 - 4E_1F_1)(A_2^2 - 4E_2F_2) - 4E_2F_1C_1^2 , \quad (\text{A8})$$

$$\Lambda = (A_1^2 - 4E_1F_1)(A_2C_2D_2 + C_2^2F_2 + E_2D_2^2) \quad (\text{A9})$$

$$+ (A_2^2 - 4E_2F_2)(A_1B_1D_1 + B_1^2F_1 + E_1D_1^2) \quad (\text{A10})$$

$$+ (D_1A_1 + 2B_1F_1)(A_2C_2 + 2E_2D_2)C_1 \quad (\text{A11})$$

$$+ C_1^2D_1^2E_2 + C_1^2C_2^2F_1 . \quad (\text{A12})$$

APPENDIX B: POSITION RESOLUTION OF ρ

In this appendix we calculate the quantity $\langle x|e^{-\xi a^\dagger}e^{\xi^* a}|y\rangle$. This is used in (2.26) to obtain the resolution of the density matrix in the position basis. From the group properties of coherent states and the Baker-Campbell-Hausdorff expansion we can get

$$\langle x|e^{-\xi a^\dagger}e^{\xi^* a}|y\rangle = \langle x|e^{-\xi a^\dagger + \xi^* a + |\xi|^2/2}|y\rangle \quad (\text{B1})$$

$$= e^{|\xi|^2/2} \langle x| \exp \left[\hat{x} \sqrt{\frac{\omega}{2\hbar}} (\xi^* - \xi) + i \frac{\hat{p}}{\hbar} \sqrt{\frac{\hbar}{2\omega}} (\xi^* + \xi) \right] |y\rangle \quad (\text{B2})$$

$$= e^{|\xi|^2/2 + (\xi^{*2} - \xi^2)/4} \exp \left[x \sqrt{\frac{\omega}{2\hbar}} (\xi^* - \xi) \right] \langle x| \exp \left[i \frac{\hat{p}}{\hbar} \sqrt{\frac{\hbar}{2\omega}} (\xi^* + \xi) \right] |y\rangle, \quad (\text{B3})$$

where we have re-written the a, a^\dagger in terms of \hat{x} and \hat{p} and again used BCH identities to dis-entangle $e^{A\hat{x}+B\hat{p}} = e^{A\hat{x}}e^{B\hat{p}}e^{-AB[x,p]/2}$. Now since $e^{-i\hat{p}x/\hbar}|y\rangle = |y+x\rangle$ the inner product on the right hand side of (B3) becomes the delta function

$$\langle x| \exp \left[i \frac{\hat{p}}{\hbar} \sqrt{\frac{\hbar}{2\omega}} (\xi^* + \xi) \right] |y\rangle = \delta(\xi_x + \sqrt{\frac{\omega}{2\hbar}}(x-y)) \sqrt{\frac{\omega}{2\hbar}}, \quad (\text{B4})$$

where $\xi_x = \text{Re } \xi$. Collecting the terms we finally get,

$$\langle x|e^{-\xi a^\dagger}e^{\xi^* a}|y\rangle = \exp \left[\frac{|\xi|^2}{2} + \frac{\xi^{*2} - \xi^2}{4} + x \sqrt{\frac{\omega}{2\hbar}} (\xi^* - \xi) \right] \delta(\xi_x + \sqrt{\frac{\omega}{2\hbar}}(x-y)) \sqrt{\frac{\omega}{2\hbar}}. \quad (\text{B5})$$

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